Baki, A.L., Lo, S.R. and Rahman, M.M. (2010) Effect of cyclic stress reversal on cyclic instability behaviour of loose sand-silt mixtures. Auckland, New Zealand: 11th IAEG Congress, 5-10 Sep 2010. 1649-1656.

Effect of cyclic stress reversal on cyclic instability behaviour of loose sand-silt mixtures

A.L. Baki & S.R. Lo
Australian Defence Force Academy, University of New South Wales, Canberra, ACT, Australia

M.M. Rahman
University of Canterbury, Christchurch, New Zealand

ABSTRACT: Cyclic liquefaction of soils with a clearly positive state parameter is a form of instability triggered by undrained cyclic loading, where instability is used in the context of continuum mechanics, i.e. a state of $d\sigma_{ij}d\varepsilon_{ij} < 0$. This paper investigates such behaviour in the frame of critical state soil mechanics (CSSM) and taking into account the influence of fines content and cyclic stress reversal. The concept of an equivalent state parameter was used in conjunction with instability stress ratio obtained from monotonic undrained tests to synthesis the cyclic tests results. Experimental results showed that instability stress ratio obtained from monotonic test at the same equivalent state parameter define the triggering of instability under cyclic loading irrespective of stress reversal. The influence of initial effective confining stresses and fines contents can be assessed from the instability stress ratio and the equivalent state parameter.

1 Introduction

Some of the worst earthquake induced damage has occurred due to liquefaction of saturated or nearly saturated sandy soils. The Niigata earthquake (1964) in Japan was an early earthquake event that brought liquefaction to the attention of engineers. Since then, liquefaction has become one of the main concerns of geotechnical engineering. Geotechnical researchers have been trying to understand and mitigate liquefaction related hazard. Significant progress has been made since 1980s in linking certain cyclic liquefaction behaviour to static instability under certain conditions. This cyclic liquefaction behaviour is a certain form of instability and will be referred to as cyclic instability in this paper. For this type of liquefaction, instability in the context of continuum mechanics, i.e. $d\sigma_{ij}d\varepsilon_{ij} < 0$, is manifested, where σ_{ij} and ε_{ij} are stress and strain tensors respectively and “d” represent infinitesimal increments. Loosely speaking, the soil is in a state of deviatoric strain softening and therefore the load/ stress carrying capacity will keep on reducing even if the load trigger causing the failure is subsequently removed. This substantial strength loss can lead to flow-like deformations of the affected soils that in turn can cause severe damage to manmade structures within the affected area.

Liquefaction can be triggered by either a single rapid monotonic stress pulse or a series of cyclic stress pulses. The former is referred to as static liquefaction whereas the later is referred to as cyclic liquefaction. Significant research since the late 80’s suggested that there are fundamental linkages between static and cyclic liquefaction. Both forms are related to generation of excess pore water pressure as a result of undrained deviatoric loading. Mohamad & Dorby (1986) reported that the monotonic behaviour of soil should be considered in analyzing undrained cyclic behaviour of saturated sands. Georgiannou et al. (1991) demonstrated that the monotonic bounding envelope obtained from undrained effective stress path (ESP) played a key role in determining cyclic behaviour of tested Ham river sand. Hyodo et al. (1994) demonstrated that cyclic instability was triggered when cyclic ESP reached the instability region of corresponding monotonic test. Gennaro et al. (2004) also demonstrated that for a given initial density, the undrained monotonic behaviour can help to predict the behaviour during undrained cyclic loading. Similar findings were reported in some other literatures (Konrad 1993, Vaid & Sivathayalan 2000, Yamamuro & Covert 2001). These studies discussed above were carried out largely on clean sand.

Sandy deposits often contain some fines and the assumption that the behaviour of such soils is similar to clean sands may be misleading (Yamamuro & Lade, 1998), and a rational basis for comparison in not often clear (Thevanayagam et al., 2002). Early observations by (Seed et al., 1983; Pitman et al., 1994) regarded the addition of fines to bring a beneficial effect to sand deposits against liquefaction, but some recent research revealed that the sand deposited with some a fines content bellow a certain limit may be more liquefiable than clean sand (Zlatovic and Ishihara, 1997; Thevanayagam et al., 2002; Yamamuro and Lade, 1998). This difference in behaviour is attributed to the micro-structural arrangement of fines and sand particles, and a variety of hypotheses have been proposed. For near-failure and/or large deformation behaviour, these hypothesis all relates back to the shift in steady state line in the e-log($p2$) space with increase in fines content, where $p2$ is the effective mean stress. This is not surprising because the SSL is the anchor concept in critical state soil mechanics framework (Been & Jefferies, 1985). For fines content, f_c, less than a certain threshold value, f_{thre}, the soil fabric is of a fines-in-sand matrix (Thevanayagam et al., 2002) and the steady state line (SSL) shifted downwards with increase in f_c. However, if $f_c > f_{thre}$, then the fabric changes to a sand-in-fines matrix and the SSL will shift upward with further increase in f_c. This is the theoretical origin of the many complicated behaviours caused by fines. This paper focused on the condition of $f_c < f_{thre}$ as this represents a wide range of sandy deposits.

Recent research by Lo et al. (2010) on cyclic instability behaviour of a sand-fines mixture with ($f_c < f_{thre}$) under one-way cyclic loading suggested the criterion for onset of instability during cyclic loading is the development of an effective stress ratio exceeding η_{IS}; where η_{IS} is the effective stress ratio at onset of instability (i.e. ESP turning downwards) under monotonic loading for a replicate specimen. However, their studies did not consider the effect of cyclic loading reversal on cyclic instability behaviour. Furthermore, it did not attempt to model the combined effects of void ratio, e, and f_c. The objective of this study is to investigate cyclic instability behaviour in relation with static instability considering cyclic stress reversal, and to capture the effects of fines content and initial conditions (prior to shearing) using the concept of equivalent state parameter recently proposed by Rahman and Lo (2007, 2009).

2 Conceptual framework for sand with fines

In this section, the conceptual framework for modelling the combined effects of fines is developed. The fact that the SSL (in the e-log($p2$) space) are dependent on f_c presents practical challenges as a new set of tests are needed to obtained the SSL. It is also theoretically undesirable as any small change in fines content implies a new soil type. The physical consequence of a fines-in-sand fabric implies that the fines are not fully effective in the force structure. Therefore, the presents of fines means that the void ratio is not an effective parameter in representing the force structure. Some form of equivalence or effective void ratio may better present the force structure.

2.1 Equivalent granular void ratio, e*

The concept of an equivalent granular void ratio, e^*, defined in Equation 1 below, was proposed by Thevanayagam et al. (2002) as a starting point in capturing the combined effects of fines and void ratio.

$$e^* = \frac{e + (1-b) f_c}{1 - (1-b) f_c}$$

where b represents the fraction of fines that actively take part in the force structure of mixed sand; and therefore 1 ε b ε 0. As discussed in Rahman et al. (2008, 2009); "b" should vary with f_c. However, b-values reported by different researchers (Chiu & Fu 2008, Ni et al., 2004, Yang et al., 2006) are averaged values (over a range of f_c)

back-analysed for their databases of undrained triaxial responses. Thus, there are issues in using e^* in a prediction unless the results are already known. To overcome this challenge, Rahman, Lo and co-workers (2008a, 2008b, 2009) developed a model based on binary packing studies so that "*b*" can be predicted from simple input parameters such as diameter ratio and f_c. Based on this model, steady state (SS) data points in e^*–log(p2) space can be described by a single trend line. This single trend line is termed as equivalent granular steady state line (EG-SSL), which is one of the main concepts used in this paper.

2.2 Equivalent state parameter

To take the concept of e^* into a predictive framework, the concept of equivalent granular state parameter is developed in this sub-section.

State parameter, ψ as originally proposed by Been & Jefferies (1985) and illustrated in Figure 1a is generally considered as a good parameter to predict the undrained instability behaviour of a sandy soil. It is pertinent to note that ψ represents combined effects of void ratio and consolidation stress, and thus capture the state of the soil prior to shearing. The higher the value of ψ, the higher is the shear-contractancy tendency. As the physical reason for instability is the generation of excess pore water pressure as a result of shear-contractancy tendency, the relationship between ψ and instability, cyclic or static, is clear. Since ψ is really a position measurement relative to SSL, and that SSL shifts with changes in f_c, we cannot compare ψ-values of soil with different fines content.

On the hypothesis that e^* is a better alternative than e, Rahman & Lo (2007, 2009) proposed the equivalent granular state parameter, ψ^*, which may be considered as a generalisation of the state parameter in order to capture the effects of fines. The ψ^* is defined by Equation 2 and illustrated in Figure 1b.

$$\psi^* = e^* - e^*_{SS}$$

where e^*_{SS} is the equivalent granular void ratio at SS. It is pertinent to note that ψ^* also capture the fines effect because to calculate ψ^*, it needs e^* and hence "*b*". In general, if the ψ^* value at start of undrained shearing, $\psi^*(0)$, is clearly positive, one can hypothesise that instability behaviour under undrained shearing, static or cyclic is likely to occur. Thus, ψ^* (0) is used in the hypothesis (will be discussed later) to predict and linkage between cyclic instability behaviour with static instability.

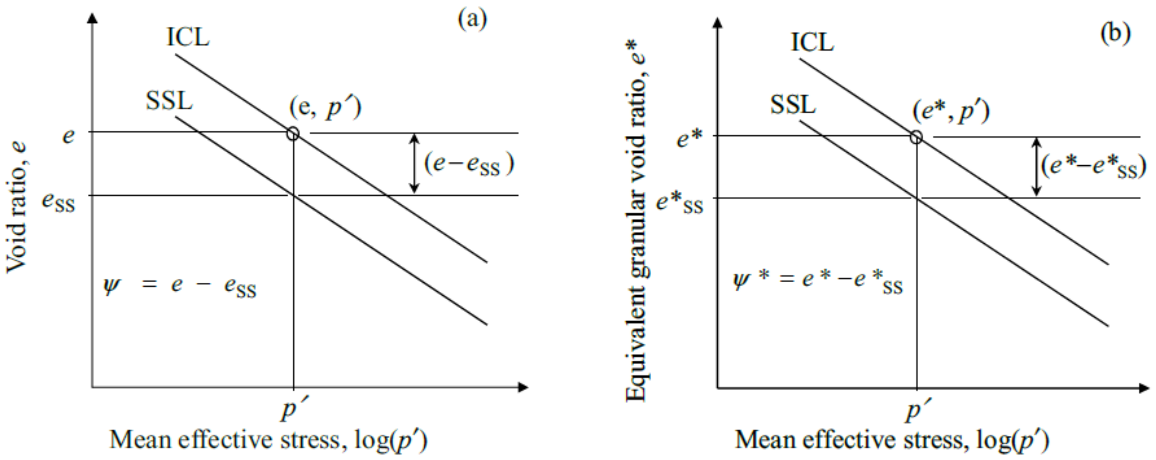

Figure 1. Definitions: a) definition of ψ (Been & Jefferies, 1985) b) definition of ψ^* (after Rahman & Lo, 2009).

3 Experimental Investigation

3.1 Testing program

A series of isotropically consolidated undrained stress controlled triaxial test were conducted on a sand with fines as described in next subsection. The fines content ranges from 0 to 30%, noting that f_{thre} for this soil is approximately 38%. Both monotonic and cyclic loading were conducted. The triaxial system allowed any combination of desired peak and trough values of cyclic stress pulses at any time throughout the test. The monotonic test results provide a benchmark for analysing cyclic instability.

3.2 Materials tested

Clean Sydney sand was used in this study. The fines added in it was a mixture of 2/3 locally available fines from Majura river bank deposits called Majura fines and 1/3 commercially available kaolin. The resultant fines has a plasticity index (PI) of 27. Four combinations of sand-fines mixture were created. The grading curve, SEM photographs and physical properties of tested materials can be found in Rahman & Lo (2008a); Lo et al. (2010).

3.3 Testing methodology

Cyclic load was applied through an external force actuator mounted at the bottom of the loading frame though deviator stress calculated using internal load cell readings. Cell pressure and pore water pressure were controlled by two digital pressure volume controllers (DPVC). A pair of internal and one external linear variable differential transformer (LVDT) were used to measure the axial deformation at the early and later stage of shearing respectively.

Cylindrical soil specimens of 100 mm diameter · 100 mm height were prepared by compacting moist tamping method in 10 uniform layers with predetermined amount of moist soil. End restraint was minimised using enlarged end platen with free ends (Lo et al., 1989). Due to the difficulties in direct measurement of sample dimensions during back pressure saturation, change in volume strain was inferred from the relation ™ε_3 H 2™ε_1 to calculate void ratio change during back pressure saturation, where ™$\sum_1$ is the change in axial strain. Details testing procedure can be found in Lo et al. (2010).

4 EQUIVALENT GRANULAR STATE PARAMETER AND instability

Based on the work of Chu & Wanatowski (2008), Chiu & Fu (2008), Yang (2002), we hypothesised that a single correlation between ψ and η_{IS}, independent of f_c, in monotonic undrained compression tests. Each monotonic test that manifest instability gave both ψ*(0) and η_{IS}, the former was calculated from the test condition and the latter from test results. This data pair can be plotted in a η_{IS} versus ψ*(0) space. The resultant plot as shown in Fig. 2 showed that the test results can be represented by a single trend curve. A bar was plotted around each due to absence of a sharp peak in the monotonic ESP. It is hypothesized that this single correlation can be used to predict onset of cyclic liquefaction through ψ*(0) as indicated by the following two steps.

1. ψ*(0) was used determined the corresponding η_{IS} value.

2. Then, a line through the origin was drawn in ESP plot of the cyclic test. This line is referred to as η_{IS} line for examining the triggering cyclic instability. Once the ESP of cyclic tests crosses this line, one may expect the triggering of cyclic instability.

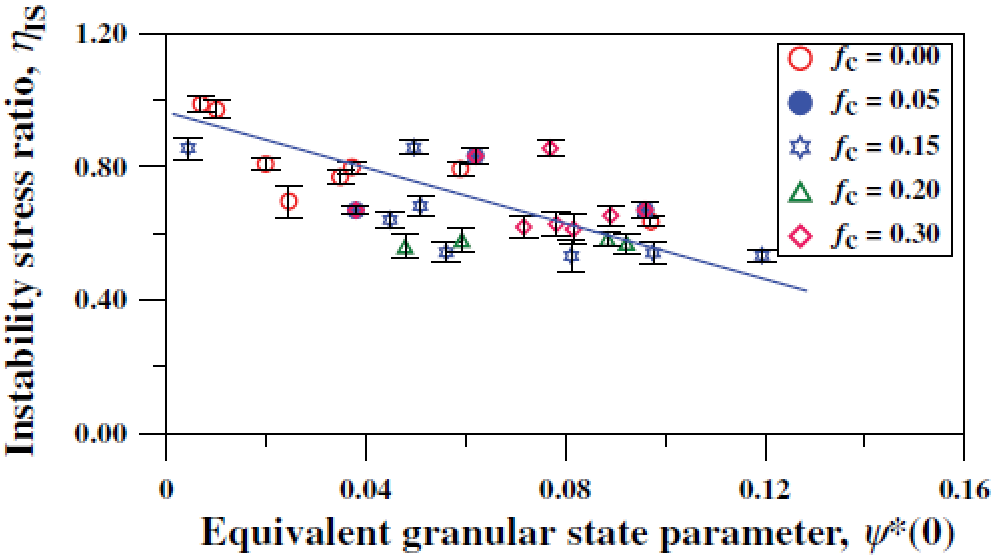

Figure 2. Relationship between η_{IS} and ψ*(0) for different % of fines.

5 Interpretation and discussion of test results

A series of triaxial one-way and two-way partial cyclic reversal loading were conducted fordifferent combinations of f_c and $p_0$2; and for different magnitude of cyclic loading pulses. Four representative tests as listed in Table 1 below are described. All four tests had ψ*(0) greater than 3 times the error band in EG-SSL and was therefore clearly positive. It is important to mention that all tests were conducted in such a way that any potential instability was triggered in compressive side of the effective stress space.

5.1 Test 1 and Test 2

These tests were designed to investigate the triggering of cyclic instability. The behaviour of two specimens of the same fines content (15%) and $p_0$2; (600 kPa) but subjected to two-way cyclic stress pulses of different magnitude was studied. Figure 3a, b represents the ESP and q-ε_1 response respectively whereas Figure 3c represents cyclic stress-time plot. Dotted line in Figure 3a–c represents test C-15-56 whereas solid line is for test C-15-57.

Test specimen C-15-56 with ψ*(0) = +0.081 was sheared applying three packets of loading pulses for different peak and trough deviator stress, denoted as q_{max} and q_{min} respectively. First packet of loading cycles had a q_{max} and q_{min} of 154 kPa and □55 kPa respectively continued for 9 loading cycles. During this period of undrained shearing, the leftward movement of ESP was getting slower. Therefore, the q_{max} was increased to 173 kPa keeping q_{min} same for next 3 cycles and ESP did not move considerably faster than previous loading pulses. Finally, q_{max} was increased to 184 kPa. Steady movement of ESP was observed up to 7th cycles of third loading pulses when ESP just touched the η_{IS} line. This η_{IS} line was drawn with η_{IS} = 0.628 obtained from Figure 2 for corresponding ψ*(0) of +0.081. After crossing the η_{IS} line; pore water pressure (pwp) developed rapidly and thus the ESP moved leftwards accordingly. Afterwards, the prescribed cyclic stress could not be achieved as indicated by point A_1 and the ESP continued to plummet downwards (Fig. 3b) even though its still in the loading phase as illustrated in Fig. 3c. Subsequently, the specimen entered the load reversal phase of the load cycle as illustrated by A_2 in Figure 3b, c. The point A_2 corresponded to an axial strain of □2.5% (tensile) but only a near-zero deviator stress could be mobilized. After A_2, it was not possible to impose the specified q_{max} or q_{min} onto the specimen. Thus, this is clearly a form of cyclic instability.

Table 1. Summary of cyclic tests conducted for this study.

Table 1. Summary of cyclic tests conducted for this study.

Test no.	Name	f_c(%)	p_0' (kPa)	e	e^*	$\psi^*(0)$	η_{IS}**	Remarks
1	C-15-56	15	600	0.654	0.896	+0.081	0.628	f_c; p_0' same
2	C-15-57	15	600	0.654	0.896	+0.081	0.628	
3	C-20-39	20	600	0.568	0.868	+0.053	0.744	f_c and p_0' different
4	C-30-61	30	350	0.487	0.896	+0.052	0.747	

** Obtained from Figure 2 corresponding to $\psi^*(0)$.

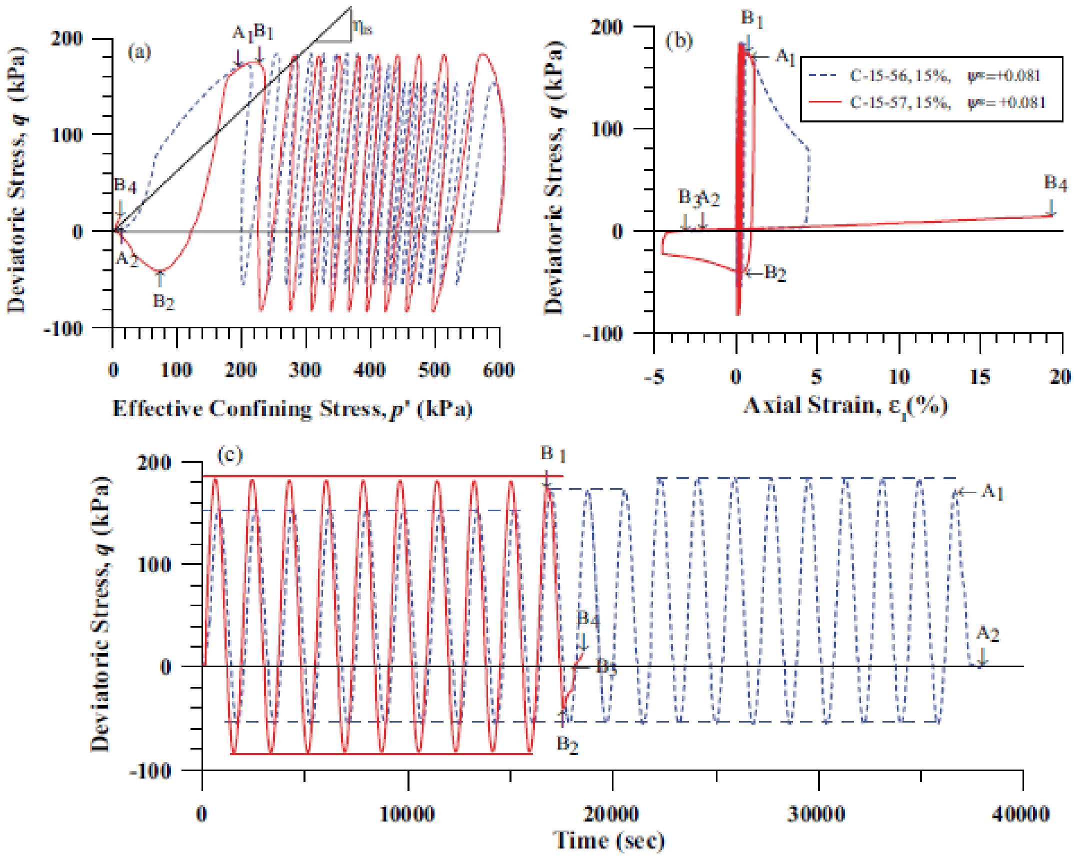

Figure 3. (a) ESP; (b) $q - \varepsilon_1$ response and (c) stress-time plot for $f_c = 15$%.

Next, another replicate sample C-15-57 sheared with only one packet of loading cycle having q_{max} of 182 kPa and q_{min} of □82 kPa. Cyclic ESP started to travel leftwards with effective stress ratio less than η_{IS} up to 7th cycle. In 8th cycle, ESP crossed the η_{IS} line same as compared with previous test and started to move faster as a result of rapid pwp generation. Thereafter, the prescribed cyclic stress could not be achieved as indicated by the point B_1 (Fig. 3c) and the ESP was continued to plummet downwards (Fig. 3a, b) as like previous test. In the following load reversal phase, the prescribed q_{min} also did not attain as indicated by the point B_2 in Figure 3c. When the ESP was returning back to the compression side from point B_3 to B_4 (Figure 3b–c); soil sample reached about 22% axial strain without attending prescribed q_{max}. Thus, this behaviour clearly indicates cyclic instability behaviour.

5.2 Test 3 and Test 4

Both tests (3 & 4) illustrated in this section were conducted under the same $\psi^*(0)$ but different f_c and p_02. Furthermore, test C-20-39 was one-way whereas test C-30-61 was two-way cyclic loading, and this provided a check on the effect of stress reversal. Figure 4a represents the ESP whereas Figure 4b represents the q□ε_1 response. Dotted line in Figure 4a, b is to represent test C-20-39 whereas solid line for C-30-61. Test C-20-39 was conducted at $f_c = 20\%$; $p_02 = 600$ kPa with $\psi^*(0)$ of +0.053. Two different packets of stress pulses were applied onto this specimen. First packet of loading cycles had a prescribed q_{max} and q_{min} of 195 kPa and 127 kPa respectively for 15 cycles. But, ESP moved very slowly leftwards in $p2–q$ space with no sign of cyclic instability. During this period, effective stress ratio was always less than η_{IS}. Then, a second packet of cyclic stress pulse with a new q_{max} and q_{min} of 255 kPa and 72 kPa was applied. Therefore, pwp started to build up gradually and hence, ESP also moved leftwards. When ESP crossed the η_{IS} line ($\eta_{IS} = 0.744$) in 6th cycle, rapid change in pwp was observed that caused ESP to move faster.

Thereafter, though the cyclic stress pulse was in loading phase, soil specimen did not attain the prescribed cyclic stress rather plummet downwards as indicated by the point C_2 shown in Figure 4a, b. In the next cycle, only 88 kPa (34%) of prescribed q_{max} was attained with huge change in axial strain (18%) when ESP travelled from point C_2 to C_3 as shown in Figure 4b. This behaviour indicated that it was not possible to attained prescribed q_{max} any more as a cause of cyclic instability.

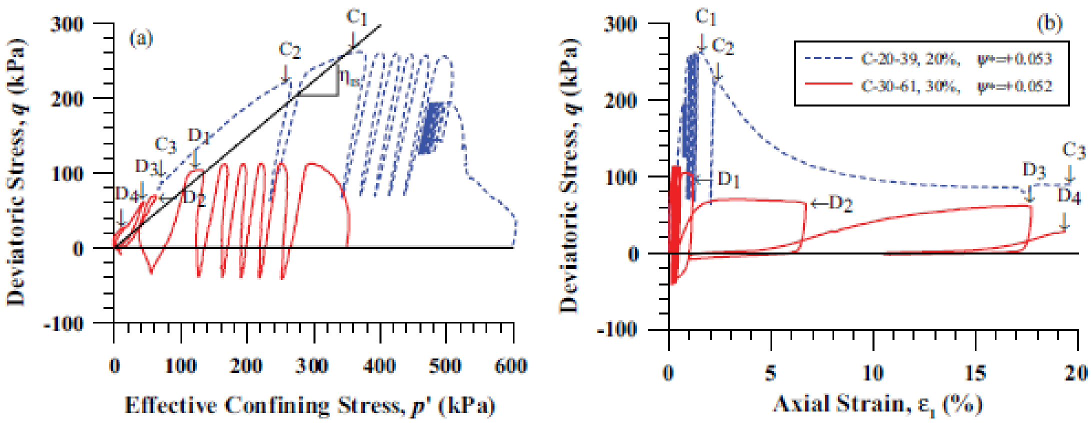

Figure 4. (a) ESP and (b) $q - \varepsilon_1$ response for different f_c and p_02.

Next, a partial reversal cyclic loading test, C-30-61 was conducted with $\psi^*(0) = +0.052$. This test was conducted at different f_c (30%) and p_02 (350 kPa) value than test C-20-39. Only one packet of loading pulse with q_{max} and q_{min} of 112 kPa and –39 kPa respectively was applied throughout the test. As soon as the ESP crossed the η_{IS} line in 6th cycle, noticeable change in both pwp and axial strain were observed. It is pertinent to note that, the same η_{IS} line as test C-20-39 was used to define triggering of cyclic instability ignoring considerable difference in $\psi^*(0)$ between two tests. The q_{max} after crossing the η_{IS} line was just less than prescribed value as indicated by D_1 in Figure 4a. However, in the next three cycles, q_{max} reduced considerably than the prescribed magnitude as indicated by D_2, D_3 & D_4 in Figure 4b. During this period, q_{min} also reduced to near-zero value. This suggests that the tested soil sample also lost its tensile deviator resistance. Hence, the observed behaviour indicates a certain form of cyclic instability. In both cases, this cyclic instability behaviour occurred within 1 cycle after crossing the η_{IS} line.

6 Conclusion

The effect of stress reversal and fines content on the triggering of cyclic instability was investigated through one-way and two-way cyclic loading tests. A hypothesis was proposed and based on this hypothesis test results were evaluated. The main concluding points from this study are:

a. There is a link between η_{IS} and $\psi^*(0)$ irrespective of fines content and initial effective confining stress.

b. There is a link between static and cyclic instability and the onset of cyclic instability can be predicted with respect to $\psi^*(0)$.

c. Loading reversal did not affect the η_{IS} value for defining the triggering of cyclic instability as long as instability was occurred in the compression side of stress space and compared with $\psi^*(0)$ for tested loose sand-silt mixture.

d. Despite cyclic instability was triggered on the compressive side, tensile deviatoric resistance was also essentially lost.

Acknowledgment

The first author would like to acknowledge University College Postgraduate Scholarship (UCPRS) for financial support while conducting his research at University of New South Wales @ADFA, Australia.

References

Been, K. & Jefferies, M.G. 1985. A state parameter for sands. *Géotechnique* 35(2): 99–112.

Chiu, C.F. & Fu, X.J. 2008. Interpreting undrained instability of mixed soils by equivalent intergranular state parameter. *Géotechnique* 58(9): 751–755.

Chu, J. & Wanatowski, D. 2008. Instability conditions of loose sand in plane strain. *Journal of Geotechnical and Geoenvironmental Engineering* 134(1): 136–142.

Gennaro, V.D., Canou, J., Dupla, J.C. & Benahmed, N. 2004. Influence of loading path on the undrained behaviour of a medium loose sand. *Canadian Geotechnical Journal* 41(1): 166–180.

Georgiannou, V.N., Hight, D.W. & Burland, J.B. 1991. Behaviour of clayey sands under undrained cyclic triaxial loading. *Géotechnique* 41(3): 383–393.

Hyodo, M., Tanimizu, H., Yasufuku, N. & Murata, H. 1994. Undrained cyclic and monotonic triaxial behaviour of saturated loose sand. *Soils and Foundations* 34(1): 19–32.

Konrad, J.M. 1993. Undrained response of loosely compacted sands during monotonic and cyclic compression tests. *Géotechnique* 43(1): 69–89.

Lo, S.R., Chu, J. & Lee, I.K. 1989. A technique for reducing membrane penetration and bedding errors. *Geotechnical Testing Journal* 12(4): 311–316.

Lo, S.R., Rahman, M.M. & Bobei, D.C. 2010. Limited flow behaviour of sand with fines under monotonic and cyclic loading. *Geomechanics and Geoengineering,* 5(1): 15–25. doi: 10.1080/17486020903452709.

Mohamad, R. & Dobry, R. 1986. Undrained monotonic and cyclic triaxial strength of sand. *Journal of Geotechnical Engineering* 112(10): 941–958.

Ni, Q., Tan, T.S., Dasari, G.R. & Hight, D.W. 2004. Contribution of fines to the compressive strength of mixed soils. *Géotechnique* 54(9): 561–569.

Pitman, T.D., Robertson, P.K. & Sego, D.C. 1994. “Influence of Fines on the Collapse of Loose Sands.” *Canadian Geotechnical Journal*, 31(5): 728–739.

Rahman, M.M. 2009. Modelling the influence of fines on liquefaction behaviour. *PhD thesis*, University of New South Wales at Australian Defence Force Academy, Australia.

Rahman, M.M. & Lo, S.C.R. 2007. Equivalent granular void ratio and state parameters for loose clean sand with small amount of fines. *Common Ground Proceedings 10th Australia New Zealand Conference on Geomechanics*, *21–24 October 2007,* Brisbane, Australia, 674–679.

Rahman, M.M. & Lo, S.R. 2008a. Effect of sand gradation and fines type on liquefaction behaviour of sand-fines mixture. *4th decennial Geotechnical Earthquake Engineering and Soil Dynamics Conference*, *GSP-181*, *18–22 May 2008,* Sacramento, USA. doi: 10.1061/40975(318)90.

Rahman, M.M. & Lo, S.R. 2008b. “The prediction of equivalent granular steady state line of loose sand with fines.” *Geomechanics and Geoengineering*, 3(3): 179–190. doi: 10.1080/17486020802206867.

Rahman, M.M. & Lo, S.R. 2009. Equivalent granular state parameter and undrained responses for sand with fines. *Proceedings of the 17th International Conference on Soil Mechanics and Geotechnical Engineering*, 5–9 October 2009, Alexandria, Egypt, 205–208. doi: 10.3233/978-1-60750-031-5-205.

Rahman, M.M., Lo, S.R. & Gnanendran, C.T. 2008. On equivalent granular void ratio and steady state behaviour of loose sand with fines. *Canadian Geotechnical Journal* 45(10): 1439–1456. doi: 10.1139/ T08-064.

Rahman, M.M., Lo, S.R. & Gnanendran, C.T. 2009. Reply to discussion by Wanatowski, D. & Chu, J. on- On equivalent granular void ratio and steady state behaviour of loose sand with fines. *Canadian Geotechnical Journal* 46(4): 483–486. doi:10.1139/T09-025.

Thevanayagam, S., Shenthan, T., Mohan, S. & Liang, J. 2002. Undrained fragility of clean sands, silty sands, and sandy silts. *Journal of Geotechnical and Geoenvironmental Engineering* 128(10): 849–859.

Seed, H.B., Idriss, I.M. & Arango, I. 1983. “Evaluation of liquefaction potential using field performance data.” *Journal of the Soil Mechanics and Foundation Division*, ASCE, 109(3): 458–482.

Vaid, Y.P. & Sivathayalan, S. 2000. Fundamental factors affecting liquefaction susceptibility of sands. *Canadian Geotechnical Journal* 37(3): 592–606.

Yamamuro, J.A. & Covert, K.M. 2001. Monotonic and cyclic liquefaction of very loose sands with high silt content. *Journal of Geotechnical and Geoenvironmental Engineering* 127(4): 314–324.

Yamamuro, J.A. & Lade, P.V. 1998. “Steady-State Concepts and Static Liquefaction of Silty Sands.” *Journal of Geotechnical and Geoenvironmental Engineering*, 124(9): 868–877.

Yang, J. 2002. Non-uniqueness of flow liquefaction line for loose sand. *Géotechnique* 52(10): 757–760.

Yang, S.L., Sandven, R. & Grande, L. 2006. Instability of sand-silt mixtures. *Soil Dynamics and Earthquake Engineering* 26(2–4): 183–190.

Zlatovic, S. & Ishihara, K. 1997. “Normalized behaviour of very loose non-plastic soils: Effect of fabric.” *Soils and Foundations*, 37(4): 47–56.